



## MOCK TEST JEE-2020 TEST-01 SOLUTION

Test Date :01-01-2020

### [PHYSICS]

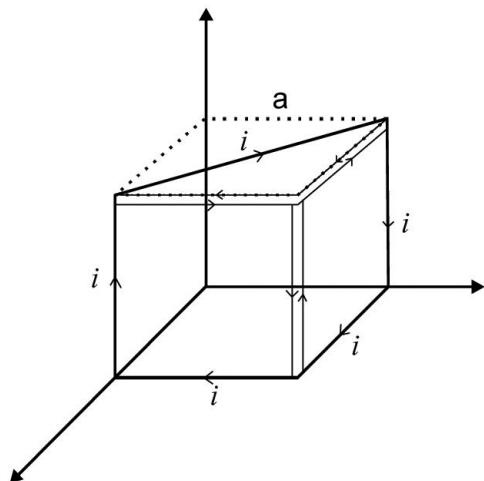
1.

**Ans. 4**

$$f_{\text{mix}} = \frac{(n\alpha)(3)(3) + (n-n\alpha)(6)}{(n\alpha)(3) + (n-n\alpha)} = \frac{3\alpha + 6}{2\alpha + 1}$$

2.

**Ans. (1)**



Here  $\vec{M} = -ia^2 \hat{i} - i \frac{a^2}{2} \hat{j} - ia^2 \hat{k}$

$$\vec{M} = -ia^2 \left( \hat{i} + \frac{\hat{j}}{2} + \hat{k} \right)$$

$$\Rightarrow |\vec{M}| = \frac{3}{2} ia^2$$

$$= \frac{3}{2} \times 3 \times 1 \quad |\vec{M}| = \frac{9}{2} \text{ ampere - meter}^2$$

3.

**Ans. (1)**

$$V_A = \frac{-GM}{3R},$$

$$V_B = \frac{-GM}{2R^3} \left( 3R^2 - \frac{R^2}{4} \right) = \frac{-11GM}{8R}$$

By conservation of energy

$$\begin{aligned} mV_A &= mV_B + \frac{1}{2}mv^2 \\ \Rightarrow \frac{-GMm}{3R} &= \frac{-11GMm}{8R} + \frac{1}{2}mv^2 \\ \Rightarrow v &= \frac{5}{2} \sqrt{\frac{GM}{3R}} \end{aligned}$$

4. (3)

5.

**Ans. (1)**

$$(\mu_1 - 1)t = n\beta$$

$$\frac{(\mu_1 - 1) \times 1.8 \times 10^{-5}}{(\mu_2 - 1) \times 3.6 \times 10^{-5}} = \frac{18\beta}{9\beta}$$

$$(\mu_1 - 1) = 4(\mu_2 - 1)$$

$$4\mu_2 - \mu_1 = 3$$

6.

**Ans. (1)**

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} \Rightarrow K = \frac{h^2}{2m\lambda^2}$$

Wavelength,

$$\lambda_0 = \frac{hc}{K} = \frac{hc}{\left(h^2 / 2m\lambda^2\right)} = \frac{2mc\lambda^2}{h}$$

7. (2)

8.

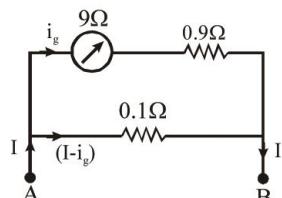
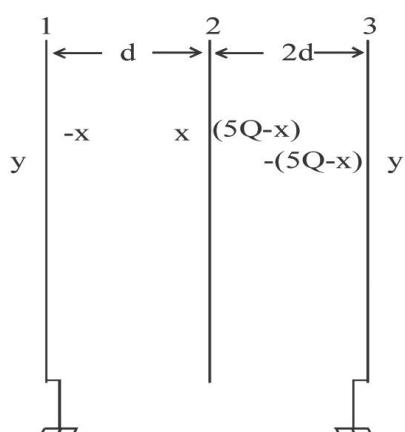
9.

**Ans. (3)**

$$i_g = 10 \text{ mA} = 0.01 \text{ A}$$

$$V_A - V_B = (I - i_g)0.1 = i_g \times 9.9$$

$$\text{or } I \times 0.1 = 10i_g$$

**Ans. (4)**

$$y = 0$$

$\because$  the potential of both the extreme plates has to be zero

$$\text{further } V_2 - V_1 = V_2 - V_3$$

$$\left(\frac{x}{A\epsilon_0}\right)d = \left(\frac{5Q-x}{A\epsilon_0}\right)(2d)$$

$$\text{or } I = \frac{10 \times 0.01}{0.1} = 1 \text{ A} .$$

10.

**Ans. (4)**

$$SL_1 - SL_2 = 10 \log_{10} \left( \frac{I_{\max}}{I_{\min}} \right)$$

$$= 10 \log_{10} \left( \frac{A_1 + A_2}{A_1 - A_2} \right)^2$$

$$\Rightarrow SL_1 - SL_2 = 20 \log_{10} \left[ \frac{\frac{A_1}{A_2} + 1}{\frac{A_1}{A_2} - 1} \right]$$

$$= 20 \log_{10} 10 = 20 \text{ dB}$$

11.

$$x = 10Q - 2x$$

$$x = \frac{10Q}{3}$$

Final charge on plate (1) is  $= -\frac{10Q}{3}$ .

Initial charge on plate (1) was  $= Q$ .

Charge flown through

$$S_1 = Q - \left(-\frac{10Q}{3}\right) = \frac{13Q}{3}$$

**Ans. (2)**

$$\text{Apparent frequency } f^1 = f_0 \left( 1 \pm \frac{u_{rel}}{v} \right)$$

$$\therefore 10 = 680 \left( 1 + \frac{u}{340} \right) - 680 \left( 1 - \frac{u}{340} \right)$$

$$\Rightarrow u = 2.5 \text{ m/s}$$

15.

12. **Ans. (2)**

Let  $x$  be the desired length

$$\text{Potential gradient in the first case} = \frac{E_0}{\ell}$$

$$\therefore E = \left(\frac{\ell}{3}\right) \cdot \left(\frac{E_0}{\ell}\right) = \frac{E_0}{3} \dots (\text{i})$$

$$\text{Potential gradient in second case} = \frac{E_0}{3\ell/2} = \frac{2E_0}{3\ell}$$

$$\therefore E = (x) \frac{2E_0}{3\ell} \dots (\text{ii})$$

From equations (i) and (ii),

$$\frac{E_0}{3} = \left(\frac{2E_0}{3\ell}\right)x \quad x = \frac{\ell}{2}.$$

13. **Ans. (1)**

$$L = \frac{nh}{2\pi} \text{ and } r \propto n^2$$

$$\Rightarrow n \propto \sqrt{r} \quad \text{so } L \propto \sqrt{r}$$

14. **Ans. (2)**

Reflection through  $M_1$

$$\frac{1}{v} + \left(\frac{-1}{15}\right) = \frac{-1}{10}$$

$$\frac{1}{v} = \frac{+1}{15} - \frac{1}{10} = \frac{2-3}{30}$$

$$v = -30 \text{ cm}$$

Reflection through  $M_2$

$$\frac{1}{v} + \frac{1}{10} = -\frac{1}{10}$$

$$\frac{1}{v} = \frac{-2}{10} \Rightarrow v = -5$$

$$M = \frac{-v}{u} = \frac{5}{10} = \frac{1}{2}$$

$$m = \frac{h_i}{h_o}, \quad \frac{1}{2} = \frac{h_i}{h_o}$$

$$\Rightarrow h_i = \frac{3}{2} \quad h_i = 1.5 \text{ cm}$$

$\therefore$  Distance of image from AB = 3 - 1.5 = 1.5 cm

**Ans. (1)**

$$\text{path difference} = \frac{yd}{D} = 900 \text{ nm}$$

Condition for missing lines

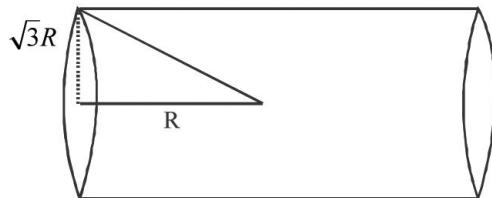
$$\text{Path Difference} = \frac{(2n-1)\lambda}{2} \Rightarrow \lambda = \frac{2\Delta x}{2n-1}$$

$$\lambda = \frac{1800}{2n-1} \text{ put } n=1,2,3$$

$$\lambda = 1800 \text{ nm}, 600 \text{ nm}, 360 \text{ nm}$$

16. (3)

17.

**Ans. (3)**

$$\tan \theta = \frac{\sqrt{3}R}{R} \quad \theta = 60^\circ$$

$$\Omega = 2\pi(1 - \cos 60^\circ) = \pi \text{ str.}$$

$$\Omega^y = 4\pi - 2(\pi) = 2\pi \text{ str.}$$

$$\phi = \frac{\Omega^1}{4\pi} \left( \frac{q}{\epsilon_0} \right) = \frac{q}{2\epsilon_0}$$

18. (2)

19. (2)

20.

**Ans. (4)**

We need to check whether they are rolling or sliding.

21. 2

22. 2

23. 2

24. 3

25. 5  $2A \sin kx = 3\sqrt{2}$

$$2 \times 3 \sin kx = 3\sqrt{2}$$

$$\sin kx = \frac{1}{\sqrt{2}}$$

$$\frac{2\pi}{\lambda} x = \frac{\pi}{4}; \frac{3\pi}{4}$$

$$x = \frac{\lambda}{8}; \frac{3\lambda}{8} \dots$$

Distance between consecutive points

$$= \frac{3\lambda}{8} - \frac{\lambda}{8} = \frac{\lambda}{4}$$

$$\frac{\lambda}{4} = 20 \text{ cm}$$

$$\Rightarrow \lambda = 80 \text{ cm}$$

$$\text{So, } (n+1) \frac{\lambda}{2} = 240$$

## [CHEMISTRY]

26.

**Ans. (3)**

Probability of finding the electron is

$$\psi^2 = 0 \quad \text{or} \quad \psi = 0 \quad \sigma = \frac{2Zr}{a_0}$$

$$(\sigma - 1) = 0 \Rightarrow \sigma = 1 \quad r = \frac{\sigma a_0}{2Z}$$

$$\text{or } r = \frac{a_0}{2Z} \quad \text{or } \sigma^2 - 8\sigma + 12 = 0$$

$$\sigma = 6; \quad \sigma = 2$$

$$\text{if } \sigma = 6 \Rightarrow r = \frac{6a_0}{2Z} = \boxed{\frac{3a_0}{Z}}$$

$$\sigma = 2 \quad r = \frac{2a_0}{2Z} = \frac{a_0}{Z}$$

$$\sigma = 1 \quad r = \frac{a_0}{2Z} = \boxed{\frac{a_0}{2Z}}$$

27.

**Ans. (3)**

Only correct match.

28.

**Ans. (4)**

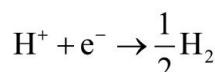
Birch reduction of aromatic ring system gives mainly unconjugated dihydroderivatives.

29. (4)

30. (4)

31.

**Ans. (2)**

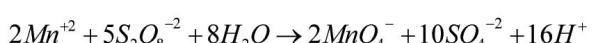


$$E = E^\circ - \frac{0.059}{1} \log \frac{1}{[H^+]} = -0.59V$$

32.

**Ans. (3)**

The product formed is  $MnO_4^-$



33. (4)

34.

**Ans. (2)**

Aromatic aldehydes that do not have  $\alpha$  hydrogen atoms on treatment with concentrated alkali undergo self oxidation and reduction to give alcohol and salt of the corresponding carboxylic acid during Cannizzaro's reaction.

35.

**Ans . (4)**

Isotones have same number of neutrons

$$17 - 9 = B - 8$$

$$B = 16$$

Isobars have same mass number

$$A = B = 16$$

$$\text{No of neutrons} = 16 - 8 = 8$$

36. (4)

37.

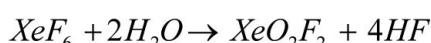
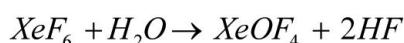
**Ans. (3)**

For monoclinic & orthorhombic, end centered unit cell is possible.

38.

**Ans. (2)**

Hydrolysis of  $XeF_6$  gives



In above two reactions there is no change in oxidation number

39. (1)

40.

**Ans. (4)**

All are valid statements for the reaction shown

41.

**Ans. (1)**

It reacts with both strong acids and strong bases.

42.

**Ans. (2)**

1 mole of XO loses =

$$\frac{1.806 \times 10^{23}}{0.1 \times 6.02 \times 10^{23}} = 3 \text{ moles of electrons}$$

43.

**Ans. (4)**

Silicons are good insulators, on methylation of dibbrane we get  $B_2H_2(CH_3)_4$

44.

**Ans. (3)**

In the first complex ligand is  $O_2^{2-}$  which is oxidised into  $O_2^{1-}$ .

hence O – O bond length decreases.

45.

**Ans . (4)**

$$P_{\text{Total}} = P_{HNO_3} + P_{NO_2} + P_{H_2O} + P_{O_2}$$

$$\therefore P_{NO_2} = 4P_{O_2} \quad \& \quad P_{H_2O} = 2P_{O_2}$$

$$\therefore P_{\text{Total}} = P_{HNO_3} + 7P_{O_2}$$

$$\Rightarrow 30 - 2 = P_{O_2} \times 7 \Rightarrow P_{O_2} = 4$$

$$K_p = \frac{(P_{NO_2})^4 \cdot P_{H_2O} \cdot P_{O_2}}{P_{HNO_3}^4}$$

$$= \frac{(4 \times 4)^4 \times (2 \times 4)^2 \times 4}{2^4} = 2^{20}$$

$$K_p = K_c (RT)^{\Delta n_g} = K_c \times (0.08 \times 400)^3$$

$$K_c = \frac{2^{20}}{(32)^3} = 32$$

46. 4 For H first excited state requires Energy change

$$= 13.6 \times Z^2 \left[ \frac{1}{1^2} - \frac{1}{2^2} \right] \text{eV} = \frac{3}{4} \times 13.6 \text{eV}$$

For  $He^+$  ion energy change after absorbing energy

$$\text{released by H-atom} = 13.6 \times Z^2 \left[ \frac{1}{2^2} - \frac{1}{n^2} \right] \text{eV}$$

$$= 13.6 \times 4 \left( \frac{1}{4} - \frac{1}{n^2} \right) = 13.6 \times \frac{3}{4}$$

$$= 1 - \frac{4}{n^2} = \frac{3}{4}$$

$$\frac{1}{4} = \frac{4}{n^2} \quad \therefore n = 4$$

47. 4 Due to isotopic effect  
2, 3, 4, 5 option are correct

48. 5 Difference in mass of compound

$$= 390 - 180 = 210$$

wt. of  $CH_3CO$  – group is = 43

$$\text{Therefore no. of } -NH_2 \text{ group} = \frac{210}{43} = 4.88 = 5.$$

49. 6 Six type of tripeptide molecules are formed.

$$50. 2 \quad (b) \quad \frac{r_{He}}{r_{CH_4}} = \sqrt{\frac{M_{CH_4}}{M_{He}}} = \sqrt{\frac{16}{4}} = 2$$

## [MATHEMATICS]

51.

**Ans. (1)**

$$f'(x) = e^{-x} (x-2)(x-4) < 0$$

52.

**Ans. (4)**

$$2 \int \frac{(\cos x + \sec x) \sin x}{(\cos^6 x + 6 \cos^2 x + 4)} dx = -2 \int \frac{(t^2 + 1)}{t^7 + 6t^3 + 4t} dt$$

Putting  $\cos x = t$ 

$$= -2 \int \frac{\frac{1}{t^5} + \frac{1}{t^7}}{1 + \frac{6}{t^4} + \frac{4}{t^6}} dt = \frac{1}{12} \ln \left( 1 + \frac{6}{t^4} + \frac{4}{t^6} \right) + C, \quad t = \cos x$$

53.

**Ans. (3)**

$$\begin{aligned} & \int_{-6}^6 \max(|2-x|, 4-|x|, 3) dx \\ &= 2 \left[ \int_0^1 (4-|x|) dx + \int_1^5 3 dx + \int_5^6 |2-x| dx \right] = 38 \end{aligned}$$

54.

**Ans. (3)**

$$BB^T = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} \frac{3}{4} + \frac{1}{4} & \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} \\ \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} & \frac{1}{4} + \frac{3}{4} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

55.

**Ans. (3)**

$$\begin{aligned} I &= \int_{\alpha}^{\beta} \frac{e^{f\left(\frac{a(x-\alpha)(x-\beta)}{x-\alpha}\right)}}{e^{f\left(\frac{a(x-\alpha)(x-\beta)}{x-\alpha}\right)} + e^{f\left(\frac{a(x-\alpha)(x-\beta)}{x-\beta}\right)}} dx \\ &= \int_{\alpha}^{\beta} \frac{e^{f(a(x-\beta))}}{e^{f(a(x-\beta))} + e^{f(a(x-\alpha))}} dx \quad \dots(1) \end{aligned}$$

$$= \int_{\alpha}^{\beta} \frac{e^{f(a(\alpha+\beta-x-\beta))}}{e^{f(a(\alpha+\beta-x-\beta))} + e^{f(a(\alpha+\beta-x-\alpha))}} dx$$

$$I = \int_{\alpha}^{\beta} \frac{e^{f(a(x-\alpha))}}{e^{f(a(x-\alpha))} + e^{f(a(x-\beta))}} dx \quad \dots(2)$$

$$2I = \int_{\alpha}^{\beta} dx \Rightarrow I = \frac{|\alpha - \beta|}{2} = \frac{\sqrt{b^2 - 4ac}}{2|a|}$$

56.

**Ans. (4)**

$$I = xf(x) \Big|_0^2 - \int_0^2 f(x) dx = 0 - \frac{3}{4} = -\frac{3}{4}$$

57.

**Ans. (4)**

$$(1+\omega)^n = {}^nC_0 + {}^nC_1\omega + {}^nC_2\omega^2 + \dots + {}^nC_n\omega^n$$

$$(1+1)^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n$$

$$(1+\omega)^n + (1+1)^n = 2C_0 + C_1(1+\omega) + C_2(1+\omega^2) + C_3(1+\omega^3) + C_4(1+\omega)$$

$$+ C_5(1+\omega^2) + C_6(1+\omega^3) + \dots + C_n(1+\omega^n)$$

$$2(C_0 + C_3 + C_6 + \dots) + (C_1 + C_4 + C_7 + \dots)(1+\omega) + (C_2 + C_5 + C_8 + \dots)(1+\omega^2) = -\omega^n + 2^n$$

$$\Rightarrow (2^n - 1) (\because n \text{ is a multiple of } 3, \omega^n = 1)$$

58.

**Ans. (2)**

Let

$$I = f\left(\frac{1}{2}\right) - f\left(\frac{1}{3}\right) = \int_0^{\pi/4} \log_e \left( \frac{1 + \frac{1}{2} \tan z}{1 + \frac{1}{3} \tan z} \right) dz = \int_0^{\pi/4} \log_e \left( \frac{3}{2} \cdot \frac{2 + \tan z}{3 + \tan z} \right) dz$$

Replacing  $z$  by  $\frac{\pi}{4} - z$ , we get

$$I = \int_0^{\pi/4} \log_e \left( \frac{3}{4} \cdot \frac{3 + \tan z}{2 + \tan z} \right) dz$$

$$\Rightarrow 2I = \int_0^{\pi/4} \log_e \left( \frac{9}{8} \right) dz = \frac{\pi}{4} \log_e \left( \frac{9}{8} \right)$$

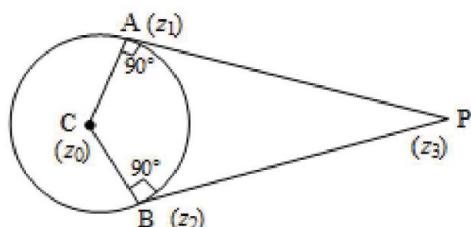
$$\Rightarrow I = \frac{\pi}{8} \log_e \left( \frac{9}{8} \right)$$

59.

**Ans. (3)**

Let  $z_1, z_2$  are represented by A, B whereas  $z_0$  is represented by C.

Let P represent  $z_3$



$$\frac{z_3 - z_1}{z_0 - z_1} = \frac{PA}{AC} e^{i\frac{\pi}{2}}, \quad \frac{z_0 - z_2}{z_3 - z_2} = \frac{BC}{PB} e^{i\frac{\pi}{2}}$$

$$\frac{z_3 - z_1}{z_0 - z_1} \cdot \frac{z_0 - z_2}{z_3 - z_2} = \frac{PA}{radius} \cdot \frac{radius}{PB} e^{i\pi} = -1 \quad (\because PA = PB)$$

60.

**Ans. (3)**

Letters of the word STATISTICS are  
AIICSSSTTT (10 letters)

Letter of the word ASSISTNAT are AAINSSSTT  
(9 letters)

Common letters are A, I, S and T

$$\text{Probability of choosing A is } \frac{1}{10} \times \frac{2}{9} = \frac{2}{90}$$

$$\text{Probability of choosing I is } \frac{2}{10} \times \frac{1}{9} = \frac{2}{90}$$

$$\text{Probability of choosing S is } \frac{3}{10} \times \frac{3}{9} = \frac{9}{90}$$

$$\text{Probability of choosing T is } \frac{3}{10} \times \frac{2}{9} = \frac{6}{90}$$

$\therefore$  Probability of required event =

$$\frac{2}{90} + \frac{2}{90} + \frac{9}{90} + \frac{6}{90} = \frac{19}{90}$$

61.

**Ans. (4)**

General equation of tangent to the curve

$$y^2 = 8x \text{ is } y = mx + \frac{2}{m}$$

Now on solving it with  $xy = -1$ , put  
discriminant = 0

$$x \left( mx + \frac{2}{m} \right) = -1 \Rightarrow m = 1$$

62.

**Ans. (1)**

$$(HHH), (RR), \underbrace{(II), (PP), AYU}_{x} = {}^{12}C_7 \cdot \frac{|7|}{[2 \cdot 2]} \cdot 1 = (198)7!$$

63.

**Ans. (3)**

$$\frac{\left(\frac{x-y+1}{\sqrt{2}}\right)^2}{10} + \frac{\left(\frac{x+y-3}{\sqrt{2}}\right)^2}{5/2} = 1$$

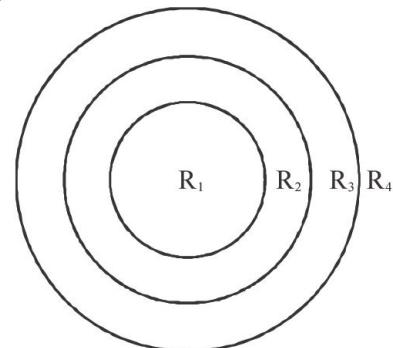
Here  $a^2 = 10$  and  $b^2 = 5/2$  and centre is (1, 2)

$\therefore$  Locus of feet of perpendicular lie on auxiliary circle of ellipse

$\therefore$  Equation of circle is  $(x-1)^2 + (y-2)^2 = 10$

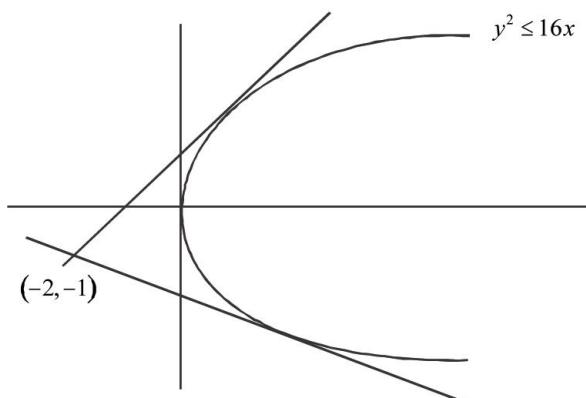
$$x^2 + y^2 - 2x - 4y - 5 = 0$$

64.

**Ans. (2)**

$$\begin{aligned} \text{Area} &= \pi \left[ (2^2 - 1^2) + (4^2 - 3^2) + \dots + (100^2 - 99^2) \right] \\ &= \pi [3 + 7 + 11 + \dots + 199] \\ &= 5050\pi \end{aligned}$$

65.

**Ans. (4)**

$S$  be the set of points inside the parabola.

$$\text{and } \frac{y+1}{x+2} \text{ is } \frac{y-(-1)}{x-(-2)}$$

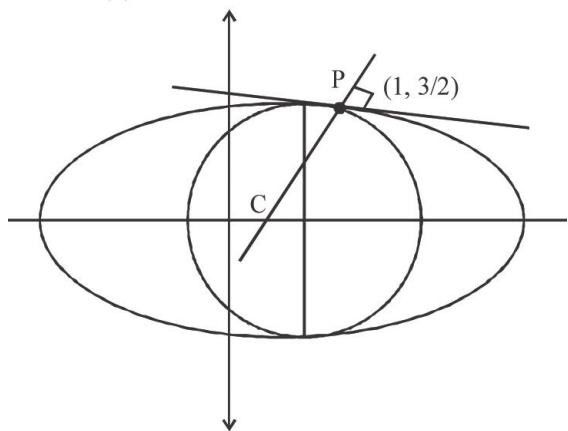
which is slope of line joining  $(x, y)$  and  $(-2, -1)$   
 $\therefore$  points  $(x, y)$  should be taken on parabola and  
then make tangents .

now eq<sup>n</sup> of tangent in slope form

$$y = mx + \frac{a}{m}$$

$$y = mx + \frac{4}{m}$$

66.

**Ans. (1)**

By symmetry centre of circle lies on X-axis

$$\therefore \text{Normal at } P \text{ is } \frac{4x}{1} - \frac{3y}{\frac{3}{2}} = 1$$

$$\Rightarrow c \equiv \left(\frac{1}{4}, 0\right)$$

$$\therefore \text{radius} = \sqrt{\left(1 - \frac{1}{4}\right)^2 + \left(\frac{3}{2}\right)^2} = \frac{3\sqrt{5}}{4}$$

$\therefore$  equation of circle is

$$\left(x - \frac{1}{4}\right)^2 + y^2 = \left(\frac{3\sqrt{5}}{4}\right)^2$$

$$-1 = -2m + \frac{4}{m}$$

$$-m = -2m^2 + 4$$

$$2m^2 - m - 4 = 0$$

$$m = \frac{1 \pm \sqrt{1+32}}{4}$$

$$m = \frac{1 - \sqrt{33}}{4} \quad M = \frac{1 + \sqrt{33}}{4}$$

$$\Rightarrow m + M = \frac{1}{2}$$

68.

$$\Rightarrow x^2 - \frac{x}{2} + y^2 + \frac{1}{16} - \frac{45}{16} = 0$$

$$x^2 - \frac{x}{2} + y^2 - \frac{44}{16} = 0$$

$$\therefore x - \text{int except} = 2\sqrt{g^2 - c}$$

$$= 2\sqrt{\frac{1}{16} + \frac{44}{16}} = \frac{2}{4}\sqrt{45}$$

$$= \frac{1}{2}3\sqrt{5} = \frac{3\sqrt{5}}{2}$$

$$y - \text{intercept} = 2\sqrt{f^2 - c}$$

$$= 2\sqrt{\frac{44}{16}} = \frac{2 \times 2}{4}\sqrt{11}$$

$$\therefore \text{product} = \frac{3\sqrt{5}}{2} \times \sqrt{11} = \frac{3\sqrt{55}}{2}$$

67.

**Ans. (3)**

No of (rectangles + squares)

$$= {}^9C_2 \times {}^9C_2 = 1296$$

and no. of squares

$$= {}^8C_1 \times {}^8C_1 + {}^7C_1 \times {}^7C_1 + \dots + {}^1C_1 \times {}^1C_1$$

$$= 1^2 + 2^2 + 3^2 + \dots + 8^2$$

$$= 204$$

$$\therefore \text{No of pure rectangles} = 1296 - 204$$

$$= 1092$$

$$\therefore \text{ratio} = \frac{1092}{204} = \frac{91}{17}$$

**Ans. (2)**A.M  $\geq$  G.M

$$\Rightarrow \frac{1+x+x^2+\dots+x^{100}}{101} \geq (1 \cdot x \cdot x^2 \cdots x^{100})^{\frac{1}{101}}$$

$$\Rightarrow \frac{1+x+x^2+\dots+x^{100}}{101} \geq x^{\frac{100+101}{2} \times \frac{1}{101}}$$

$$\Rightarrow \frac{1+x+x^2+\dots+x^{100}}{101} \geq x^{50}$$

$$\Rightarrow \frac{1}{101} \geq \frac{x^{50}}{1+x+x^2+\dots+x^{100}}$$

$$\therefore \text{Greatest value} = \frac{1}{101}$$

69.

**Ans. (1)**

$$\text{Expression} = x^2 + 2y^2 + 4x = 9 + y^2 + 4x$$

$$= 9 + 9\sin^2 q + 4(3 \cos q) = 9 + 9(1 - \cos^2 q) + 12\cos q$$

$$= 18 - 3[3\cos^2 q - 4\cos q]$$

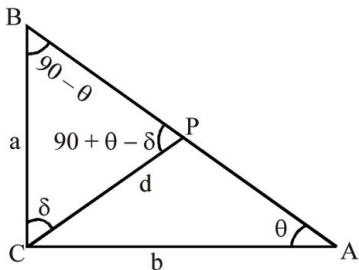
$$= 18 - 3 \times 3 \left[ \cos^2 \theta - \frac{4}{3} \cos \theta \right]$$

$$= 18 - 9 \left[ \left( \cos \theta - \frac{2}{3} \right)^2 - \frac{4}{9} \right]$$

$$= 18 + 4 - 9 \left( \cos \theta - \frac{2}{3} \right)^2 = 22 - 9 \left( \cos \theta - \frac{2}{3} \right)^2$$

Clearly maximum value = 22

70.

**Ans. (3)**

We have  $\frac{a}{\sin(90^\circ + \theta - \delta)} = \frac{d}{\cos \theta}$  (By sine rule in  $\triangle BCP$ )

$$\Rightarrow \frac{a}{d} = \frac{\cos(\theta - \delta)}{\cos \theta} = \frac{\cos \theta \cos \delta + \sin \theta \sin \delta}{\cos \theta}$$

$$\Rightarrow \frac{a}{d} = \cos \delta + \tan \theta \sin \delta \quad \dots(1)$$

$$\text{or } \frac{1}{d} = \frac{\cos \delta}{a} + \frac{\sin \delta}{a} \tan \theta$$

$$\text{But } \tan \theta = \frac{a}{b}$$

$$\Rightarrow \frac{1}{d} = \frac{\cos \delta}{a} + \frac{\sin \delta}{a} \left( \frac{a}{b} \right) \dots \quad \text{Hence } \frac{1}{d}$$

$$= \frac{\cos \delta}{a} + \frac{\sin \delta}{b}$$

$$71. \text{ Let } P(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + a_3 x^{n-3} + \dots + a_n$$

$$P(2x) = a_0 2^n x^n + a_1 2^{n-1} x^{n-1} + \dots + a_n$$

$$P(x) + P(2x) = 5x^2 - 18$$

$$\Rightarrow (a_0 + a_0 2^n)x^n + \dots + 2a_n = 5x^2 - 18$$

$$\therefore n = 2$$

$$(a_0 + 4a_0)x^2 + 2a_2 = 5x^2 - 18$$

$$5a_0 = 5 \text{ and } a_2 = -9$$

$$\therefore P(x) = x^2 - 9$$

$$\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} = \lim_{x \rightarrow 3} (x + 3) = 6$$

72.

$$px^2 + qx + r = 0$$

$$rx^2 + qx + p = 0$$

$$\therefore x = -1$$

$$\therefore p - q + r = 0$$

73.

$$|1| + |2| + \dots + |n| = 9 + 24P \quad [n \geq 4]$$

74.

Eccentricity of the hyperbola  $x^2 - y^2 \sec^2 \theta = 5$  is

$$e_1 = \sqrt{\frac{1 + \sec^2 \theta}{\sec^2 \theta}} = \sqrt{1 + \cos^2 \theta}$$

Eccentricity of the ellipse  $x^2 \sec^2 \theta + y^2 = 25$  is

$$e_2 = \sqrt{\frac{\sec^2 \theta - 1}{\sec^2 \theta}} = |\sin \theta| \quad \text{Given } e_1 = \sqrt{3}e_2$$

$$\Rightarrow 1 + \cos^2 \theta = 3 \sin^2 \theta \Rightarrow \cos \theta = \pm \frac{1}{\sqrt{2}}$$

$\therefore$  Least positive value of  $\theta$  is  $\frac{\pi}{4} \therefore P = 4 \Rightarrow 2P = 8$

75.

As  $x, y \in R$  and  $xy > 0$ , so  $x$  and  $y$  will be of same sign.

All the quantities  $\frac{2x}{y^3}, \frac{x^3y}{3}, \frac{4y^2}{9x^4}$  are positive.

A.M.  $\geq$  G.M.

$$\Rightarrow \frac{2x}{y^3} + \frac{x^3y}{3} + \frac{4y^2}{9x^4} \geq 3 \left( \left( \frac{2x}{y^3} \right) \left( \frac{x^3y}{3} \right) \left( \frac{4y^2}{9x^4} \right) \right)^{\frac{1}{3}}$$

$$= 3 \times \frac{2}{3} = 2$$